JOURNAL OF FINANCIAL INTERMEDIATION 2, 95–133 (1992)

Debt Covenants and Renegotiation*

MITCHELL BERLIN

Leonard N. Stern School of Business, New York University, New York, New York 10006

AND

LORETTA J. MESTER

Federal Reserve Bank of Philadelphia, Philadelphia, Pennsylvania 19106-1574

Received October 29, 1991

We analyze the value to firms of being able to renegotiate covenants in their debt contracts. Covenants control agency problems, but also reduce firms' flexibility to pursue profitable investments. Initial covenants will be more severe for renegotiable contracts, because they can be relaxed selectively when the lender believes they pose an inefficient constraint. We show firms with high ex ante credit risk find the option to renegotiate most valuable. The model is used to explain why bank loans and privately placed debt typically have harsher covenants than public debt and to predict which firms will borrow using closely held debt. *Journal of Economic Literature* Classification Numbers: D82, G21, G32.

1. INTRODUCTION

Why do some firms place their debt publicly and others place their debt privately or rely on bank loans? We suggest that the comparative ease with which private placements and bank loans can be renegotiated is a

* We thank Doug Bernheim, David Brown, Mark Flannery, Kose John, George Mailath, George Pennacchi, Tony Saunders, Itzhak Swary, Greg Udell, participants of the 1990 Hilliard-Lyons Symposium on the Design of Contracts, Institutions, and Markets at Indiana University, two anonymous referees, and the editor for helpful comments. Berlin thanks the Leonard N. Stern School of Business for financial support during the summer of 1990. The views expressed here are ours and do not necessarily represent the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

1042-9573/92 $5.00

major factor in a firm's choice, when renegotiability is valuable to the borrowing firm. Thus, factors that affect the value of the option to renegotiate debt should be related to a borrower's debt choice.

Private placements and bank loans typically have more stringent and detailed restrictive covenants then publicly placed bonds, a connection that Smith and Warner (1979) have argued arises partly from the greater ease of renegotiating contract terms with a small number of well-informed institutional investors. Such renegotiations are a routine part of private debt financings; Zinbarg (1975) reports that loans in Prudential Insurance Company of America's portfolio are renegotiated once a year on average—and are not merely a response to borrowers' distress. After providing a formal analysis of the connection between the severity of contractual covenants and the ease of renegotiation, we show that the value of being able to renegotiate contractual provisions depends on the severity of agency problems. In particular, we show that firms that are poor ex ante credit risks find the option to renegotiate valuable and that as the firms' creditworthiness rises (above some critical level), the ex ante value of the option to renegotiate the covenants decreases. The theory suggests that poorer credit risks will depend more on bank loans and private placements, since these are easier to renegotiate. This is consistent with the empirical evidence.

We explore a simple model in which debt contacts may be written with covenants of varying degrees of restrictiveness. The purpose of these covenants is to reduce agency problems, in particular, to reduce the borrowing firm's incentive to underinvest in activities that raise firm value when market conditions are difficult. Since these market conditions are also those in which ultimate default is most likely, firm insiders have an incentive to shift resources toward high-risk activities. To reduce borrowing costs, borrowing firms will often agree to restrictions written into the debt contract that are intended to counteract this incentive. But these restrictions are not without cost. When market conditions are good, firm insiders have less incentive to expropriate creditors, and the covenant restrictions can reduce managers' flexibility to make profitable investments. In theory, complex, contingent bond indentures might avoid these trade-offs, but realistic covenant restrictions take the form of simple rules which strike a balance between the control of agency problems and the loss of flexibility.

In addition to designing the optimally restrictive covenant, we permit borrowers to choose between contracts that permit renegotiation of the contract and contracts for which renegotiation is impossible. On one interpretation, this is a stylized way of capturing a borrower's choice between bank loans and private placements, on the one hand, and public debt, on the other. For both bank loans and private placements, the number of investors is small, reducing the transactions costs and holdout problems that arise in attempting to fashion a bargaining coalition. In addition, the investors tend to be relatively well informed, which avoids some of the well-known problems of bargaining when information is asymmetric. Government regulations also increase the cost of renegotiating public debt—the Trust Indenture Act of 1939 requires that two-thirds of debtholders agree to any changes in the indenture (Leftwich, 1981). There is empirical support for the argument that closely held debt is comparatively easy to renegotiate. In addition to Zinbarg (1975) cited above, almost half of the bank credit agreements that Lummer and McConnell (1989) identified from announcements in the *Wall Street Journal* are revisions to existing credit agreements rather than new ones.

When covenants may be renegotiated selectively, contingent upon the lender's receipt of favorable information about the firm's prospects, initial covenants will optimally include more stringent covenants.[1] But the relative attractiveness of a policy of reducing the firm's unilateral discretion and providing flexibility through renegotiation will depend upon the ex ante creditworthiness of the firm and upon the quality of the interim information available to lenders.

We show that the *value of the option to renegotiate*—the difference in the borrower's expected profit under a contract when renegotiation is possible and when it is impossible—is high when the firm's ex ante creditworthiness is low. This result is supported by the empirical research of Blackwell and Kidwell (1988), who find that issuers of speculative-grade debt would have paid significantly more had they sold their debt publicly instead of privately. After some initial range in which it may be increasing, the value of the option to renegotiate decreases as the firm becomes more creditworthy. In essence, the benefit of writing very stringent covenants decreases as agency problems become less severe; hence the value of the option to selectively renegotiate the contract declines. In addition, we find that the option to renegotiate increases in value as the quality of the lender's information increases.

The paper proceeds as follows. Section 2 presents a brief review of the relevant literature. After presenting the model in Section 3, we derive the optimal debt contracts with and without renegotiation in Section 4. In Section 5 we present and interpret the main results: our model's empirical predictions concerning the value of the option to renegotiate and firm creditworthiness. Section 6 presents the conclusion and contains a discussion of directions for further research.

[1] DeAngelo *et al.* (1990) provide evidence that private debt covenants are set tighter than those in public debt contracts.

2. A BRIEF REVIEW OF THE LITERATURE

Jensen and Meckling (1976) first proposed viewing covenants as bonding activities, and Smith and Warner's (1979) conjecture that the stringency of covenants in private placements stems from the ease of renegotiation forms the starting point for our study. More recently, theoretical accounts of optimal covenant restrictions include Barnea *et al.* (1985), Berlin and Loeys (1988), John and Kalay (1982, 1985), and Stulz and Johnson (1985). John and Kalay's (1982) model of optimal payout constraints as a solution to the underinvestment problem is closest in spirit and structure to our own model. Like our model, John and Kalay adopt an incomplete contracting approach, and their dividend payout constraint places a floor on the borrower's investment in some activity. However, they do not analyze the factors affecting the stringency of covenants nor the possibility of renegotiation, the main concerns of our paper.

Recently, a number of papers have analyzed models of financial contracting where renegotiation is a central element. Aghion and Bolton (1988) view covenants as a means of optimally allocating control rights between managers and creditors and show that, if the borrower faces liquidity constraints, then renegotiation need not achieve the first best, even when information is symmetric. The central contractual trade-off in Aghion and Bolton, controlling agency problems versus giving the borrower discretion to make efficient decisions, is similar to our main concern. But they do not analyze the implications of the option to renegotiate for the optimal design of initial contracts. Also, while their model of optimal control regions based on noisy indicators suggests factors that may be important in determining the optimal stringency of covenants, they do not pursue this question.

Huberman and Kahn (1988a) show that the threat of bankruptcy can increase efficiency even though both borrower and lender know that contract renegotiation will always prevent bankruptcy. This effect also occurs in our model, where the option to renegotiate permits contracts to utilize more stringent covenants, which are then renegotiated selectively. Unlike our model, Huberman and Kahn's contracts with renegotiation always achieve first best, and when renegotiation is valuable, it occurs with probability one. These conclusions follow because their model has neither uncertainty about borrower type nor asymmetric information, the two elements that drive our comparative statics results concerning the value of the option to renegotiate.[2] When there is uncertainty and a cost to contract complexity, Kahn and Huberman (1989) show that renegotiation can achieve an efficient outcome while no other simple contract can.

[2] Hart and Moore (1989) and Sharpe (1988) also present financial contracting models in which renegotiation is a central issue. Neither paper considers the possibility that the renegotiability of the contract may be an object of design. In turn, neither examines the value of the option to renegotiate.

Information	Period	Action
Borrower and lender know prior p	0	Contract is signed Lender invests k in monitoring apparatus when renegotiation is possible
Borrower learns: (1) Type s (2) Whether lender will be informed	1	Borrower chooses x_j^i
Lender learns borrower type with probability θ	2	Borrower may propose new contract if renegotiation is possible Lender may accept or refuse Borrower is audited and: (1) "Liquidated" if breach (2) Permitted to continue if no breach
State of market z is realized	3	Revenues are produced

FIGURE 1

3. THE MODEL

A. *Timing and Information Structure*

A risk-neutral entrepreneur, the borrower, has no wealth and needs to borrow one dollar of funds from a risk-neutral lender at time 0 to carry out a project that pays off in period 3.[3] The market for loanable funds is competitive, and lenders have a required (three-period) factor of d.

The following discussion of the timing of production and the information structure of the model is illustrated in Fig. 1.

The revenue that the borrower can produce in period 3 depends upon the state of the market, a random variable $z \in \{z_b, z_g\}$, which is realized in period 3 just before revenue is actually produced. This random variable summarizes conditions affecting the firm's profitability, for example, the aggressiveness of the firm's competitors, the quality of the firm's management, the costs of the firm's inputs, or the demand for the firm's products. The prior probability that $z = z_g$ and the market is good is p, while the prior probability that $z = z_b$ and the market is bad is $1 - p$. This prior

[3] Assuming risk neutrality is not essential. In our model, the borrower's payoffs are more variable when renegotiation is possible, so a risk-averse borrower would find the option to renegotiate the contract less valuable than a risk-neutral borrower.

probability is known to both the borrower and lender in period 0 and may be viewed as a measure of the firm's ex ante creditworthiness.

At $t = 1$, only the borrower receives some news, $s \in \{s_b, s_g\}$, which provides information about the ultimate state of the firm's market. Conditional upon this information, the borrower makes an updated forecast, $p_{ij} \equiv p(z_i|s_j)$, the probability that the state of the market will be z_i given news s_j. We assume that the news contains a small amount of noise, so that $p_{gg} = p_{bb} = 1 - \varepsilon$, and $p_{gb} = p_{bg} = \varepsilon$. Since the news is very informative about the likely state of the market, we often refer to a borrower that observes s_g as a good borrower and one that observes s_b as a bad borrower.

As an insider, the borrower is likely to be the first to become aware of any important information about firm prospects. But a lender that has borne the initial costs of developing expertise about market conditions will ultimately be able to identify and interpret at least some types of events. Essentially public events, like entry into the firm's market, are of this type. Other events, like the results of the firm's initial marketing efforts, are easy for the borrower to misrepresent, so even a relatively well-informed lender will remain at an informational disadvantage. Since the type of information determines whether the lender becomes informed, the borrower is likely to know whether the lender will become informed.

We formalize these ideas as follows: In period 0, the lender may bear some fixed cost k which permits it to observe the signal s *in period 2* with probability θ.[4] That is, with probability θ, s is public information and the lender and borrower become symmetrically informed in period 2. We refer to this state as the *informed state*. With probability $1 - \theta$, the borrower's information remains private, and we refer to this state as the *uninformed state*. When the borrower observes s in period 1, it also knows whether the lender will be informed in period 2.

Finally, we assume that final revenues are unobservable and that the optimal contract is a debt contract.

B. *The Borrower's Production Decision*

Immediately upon receiving the news in period 1, the borrower must make a final decision about how funds will be allocated between two types

[4] The lender will not find it in its interest to bear this cost if the contract does not permit renegotiation. We do not endogenize the lender's decision to monitor, and it is crucial for our results that the borrower knows that the lender has borne the monitoring cost if the contract is renegotiable. It would not be difficult to endogenize the lender's decision, but the profitability of monitoring will depend crucially upon the relative bargaining power of the borrower and lender in contract renegotiations. We have chosen to avoid this interesting issue, because we do not have a convincing theory of bargaining power in contract renegotiations. This is an interesting direction for future research.

of activities. Although certain activities certainly increase firm value whether market conditions are good or bad, others are targeted to particular market conditions: Consider, for example, the producer of an innovative desktop computer. If the producer expects large sales, an investment that allows it to supply buyers of the desktop computer with peripherals may be profitable because of the customer base that will be established. However, when sales are expected to be modest, the same investment would reduce firm value because the opportunities for related sales will be low. In a stagnant market, a firm's revenues might be increased by more attentive billing and inventory control procedures, by reducing the workforce, or by hoarding cash. Where profitable growth opportunities exist, these measures would divert resources from more productive uses.

To formalize this idea, we classify productive activities into those that increase revenue when market conditions are good and those that increase revenue when market conditions are bad. For simplicity we assume that this classification is a partition, so any investment that increases revenue in a good market has no effect on revenue in a bad market and vice versa.[5] Accordingly, let x_j^i denote the investment in an activity that increases revenue in state of the market z_i taken by a borrower who has observed news s_j. We refer to the activities that increase revenue in good market conditions as *growth activities* and the activities that increase revenue in bad market conditions as *safety activities*. Let the borrower's revenue in a good market be denoted by $R^{\mathrm{g}}(x_j^{\mathrm{g}}, x_j^{\mathrm{b}})$ and the borrower's revenue in a bad market be denoted by $R^{\mathrm{b}}(x_j^{\mathrm{g}}, x_j^{\mathrm{b}})$. Then our assumption that particular types of activities are productive only in particular states means that

$$R^{\mathrm{g}}(x_j^{\mathrm{g}}, x_j^{\mathrm{b}}) = R^{\mathrm{g}}(x_j^{\mathrm{g}}, 0) \quad \text{and} \quad R^{\mathrm{b}}(x_j^{\mathrm{g}}, x_j^{\mathrm{b}}) = R^{\mathrm{b}}(0, x_j^{\mathrm{b}}) \quad \forall x_j^{\mathrm{g}}, x_j^{\mathrm{b}}. \tag{1}$$

For ease of notation we suppress the unproductive argument in each revenue function and write

$$R^{\mathrm{g}}(x_j^{\mathrm{g}}) \equiv R^{\mathrm{g}}(x_j^{\mathrm{g}}, x_j^{\mathrm{b}}) \qquad \text{and} \qquad R^{\mathrm{b}}(x_j^{\mathrm{b}}) \equiv R^{\mathrm{b}}(x_j^{\mathrm{g}}, x_j^{\mathrm{b}}).$$

Letting $R_x^i(x)$ and $R_{xx}^i(x)$ denote the first and second derivatives, the revenue function for state of the market z_i, $i = \mathrm{g}, \mathrm{b}$, is increasing and concave in x_j^i, investment targeted at state of the market z_i, that is

$$R_x^i(x_j^i) > 0 \qquad \text{and} \qquad R_{xx}^i(x_j^i) < 0. \tag{2}$$

[5] The assumption that activities that increase revenue in the good market have no effect on revenue in the bad market, and vice versa, is stronger than we need. Since all of our results are implied by the concavity of the revenue functions, all we require is that there not be too much complementarity between the activities.

We assume that in period 1, the full one dollar of funds initially invested in the firm may be allocated between the two activities. So the borrower who has observed s_j confronts the following resource constraint:

$$x_j^{\mathrm{g}} + x_j^{\mathrm{b}} \leq 1. \tag{3}$$

Given this resource constraint, the following conditions generate a contracting problem:

$$(1 - \varepsilon)R_x^{\mathrm{b}}(1) > \varepsilon\, R_x^{\mathrm{g}}(x), \qquad \forall x, \tag{4}$$

$$(1 - \varepsilon)R_x^{\mathrm{g}}(1) > \varepsilon\, R_x^{\mathrm{b}}(x), \qquad \forall x, \tag{5}$$

$$R^{\mathrm{b}}(1) < d, \tag{6}$$

and

$$R^{\mathrm{g}}(0) > d. \tag{7}$$

Condition (4) says that it is efficient for the bad borrower to invest solely in safety activities, because the probability of a good market is very low for a borrower that has learned bad news. Condition (5) says that it is efficient for the good borrower to invest solely in growth activities, because the probability of a bad market is very low for a borrower that has learned good news. Condition (6) implies that, despite condition (4), the bad borrower has no incentive to invest in safety activities, since the maximum revenue in the bad market is lower than the minimum possible loan rate. Finally, condition (7) guarantees that there will be loan rates such that a borrower in a good market can feasibly repay the loan. We discuss the implications of relaxing these conditions in Section 4.

C. *Covenants*

Consider now the available contractual options. Since final revenues are unobservable, we restrict attention to debt contracts, which specify that the borrower make some fixed payment r in period 3. If the borrower is unable to make the contractual payment, the lender lays claim to all revenue that is produced.[6] A key feature of our model is that the lender can exercise *limited* control over the borrower through the use of a contractual covenant.

In general, it will be prohibitively difficult to guarantee efficient investment decisions by the borrower. This would require a bond indenture with

[6] Although we make no assumption about the costs of bankruptcy if the borrower defaults on third-period loan payments, the firm's revenue in the bad state can be interpreted as net of any bankruptcy costs.

an excessively detailed menu of permissible investment choices, each contingent on a full description of prevailing market conditions. There is a high cost for such complexity. And even if a lender has the expertise to recognize an imprudent investment (given the prevailing market conditions), it would have to be able to demonstrate to a court's satisfaction that the activities were, in fact, proscribed by the contract. Verifiability is difficult. Thus, such detailed, intrusive contracting practices are simply infeasible in practice.[7]

This does not mean that real world lenders must simply cede total control to the borrower. In practice, debt contracts include indentures that target certain key *measurable* activities that are not too difficult to monitor and that are unambiguous to a comparatively uninformed third-party enforcer like the courts. Such covenants include restrictions against the sale of assets, limits on dividends, and the requirement that firms satisfy certain financial ratios indicative of financial health.[8] Of course, a firm may be able to satisfy the letter of the contract while acting inefficiently. But the firm's capacity to do so is limited as long as it honors the contract.

To model this kind of partial control through restrictive covenants, we assume that contracts may be conditioned on an indicator that is strictly increasing in the level of safety activities undertaken by the firm, $I(x_j^b)$, where $I_x > 0$.[9] This indicator is produced costlessly as a by-product of the normal accounting practices of the borrower, cannot be falsified, and is easily verified by the courts. Since the indicator is strictly increasing and the units are arbitrary, without loss of generality we assume that $I(x_j^b) = x_j^b$.

This indicator might be a measure of working capital or current assets. The aggressive pursuit of growth opportunities—producing new products or expanding into new markets—will typically involve immediate increases in the fixed capital stock and increases in expenses. This will lower balance sheet indicators of liquidity and, in turn, the value of the indicator would be low. Simply forgoing growth activities and allowing revenues to accumulate within the firm, or cutting variable costs, will tend to increase indicators of liquidity. On a second interpretation, exploiting the most profitable growth opportunities in a good market might necessitate acquisitions and liquidations. On the other hand, asset substitution in a bad market might reduce the expected value of the firm's assets. Thus, growth opportunities are restricted and the value of the firm's assets in

[7] See Huberman and Kahn (1988a) for a nice discussion of contractual complexity and verifiability.

[8] See Smith and Warner (1979) for an exhaustive account of bond indentures. Castle (1980) provides a list of common covenants in term loans.

[9] Alternatively, we may view the indicator as strictly decreasing in the growth activities undertaken by the firm.

bad markets is increased by limits on asset substitution. On this interpretation, the indicator would be inversely related to the dollar value of asset sales.[10]

Covenant restrictions stipulate that the borrower will be audited in period 2 and that it is in compliance if $x_j^b \geq x^*$, where x^* is the minimum acceptable level of the safety activity.[11] So the covenant might be a minimum working capital requirement or a restriction on asset sales. Castle's (1980) list of covenant restrictions in term loan agreements indicates that such restrictions are common. In addition, his list includes a requirement to engage only in the present line of business, which can be interpreted in the model as a limit on the firm's growth activities.[12]

We assume for now that the courts can enforce the following type of contract: If no breach has occurred, then the borrower retains control of the firm until period 3. If the covenant is breached, then control of the firm is transferred to the lender, which liquidates the firm's assets and receives this liquidation value in period 3. Later we present conditions that guarantee that the lender would wish to liquidate in the event of breach. The borrower is assumed to receive no share of the firm's value in liquidation, and we make the following assumptions about the liquidation value of the firm, L^j, j = g, b.[13]

First, the liquidation value of the firm in a bad market is greater than any revenues it might produce in period 3, or

$$R^{\mathrm{b}}(1) < L^{\mathrm{b}}. \tag{8}$$

Second, the maximum revenue producible in period 3 by the firm in a good market is greater than its liquidation value, or

$$R^{\mathrm{g}}(1) > L^{\mathrm{g}}. \tag{9}$$

[10] On a third interpretation, the indicator might refer to revenues produced in the second period. Thus, our model of the optimal stringency of restrictive covenants might be reinterpreted as a model of debt maturity.

[11] Without loss of generality, we assume the cost of the audit is zero.

[12] In our model, the indicator of the level of safety activities is not noisy. Noisy indicators would generate another cost of using stringent covenants and provide another rationale for renegotiable contracts. When the contract is renegotiable, an informed lender can avoid liquidating good firms that unwittingly breach. The optimal stringency and the relative attractiveness of renegotiable contracts will depend not only upon the ex ante creditworthiness of the firm but also upon the precision of the indicator. In a somewhat different model, Berlin and Loeys (1988) show that renegotiable contracts are attractive when indicators are imprecise and the ex ante creditworthiness of the firm is neither very low nor very high.

[13] We assume for simplicity that the firm's liquidation value is independent of its investment decision. While this assumption is somewhat extreme, we might imagine that the value of the firm's assets in alternative uses is largely independent of their efficient or inefficient deployment in the borrower's own market.

Assumption (8) conveys the plausible idea that in a bad market the firm's assets might more profitably be deployed elsewhere, and a wholesale liquidation of the firm's assets would be desirable. On the other hand, it would be preferable to permit a firm in a good market to continue production rather than sell off the firm's assets. Although it is a convenient shorthand to refer to L^g as the good firm's liquidation value, we might just as well imagine that a formal breach of contract leads to a costly, public reorganization. Assumption (9) implies that both borrower and lender have a common interest in avoiding a formal breach of contract when both know that market conditions are likely to be good.[14]

Finally, we make the following parametric restriction:

$$pR^g(1 - x) + (1 - p)R^b(x) < pL^g + (1 - p)L^b, \qquad \forall x. \tag{10}$$

Interpreted literally, assumption (10) implies that based upon prior probabilities, a policy of liquidating the firm in period 1 maximizes the firm's expected value. But this implication is an artifact of the stylized nature of the model and would not follow if the model were changed in a simple way that would leave all of the model's predictions intact but would add some cumbersome notation: Consider the addition of states in which the covenant is not binding on the good firm. Then properly interpreted, p is the probability that the firm is good *conditional* on the covenant's being binding.[15]

The full import of assumptions (8)–(10) becomes clear in Section 4 where we discuss contract renegotiations in detail. However, it may be worth mentioning now that, in equilibrium, firms are never liquidated in period 2; either borrowers satisfy the original contract or the covenant restriction is renegotiated. Production decisions and bargaining outcomes are affected by the threat of liquidation, but liquidation never actually occurs in equilibrium.

[14] In his account of loan workouts for institutional investors, Rosenberg (1984) discusses the benefits of avoiding formal breach. On the one hand, the firm's other creditors will often have cross-default clauses or contractual protections requiring notification of a breach of any of the firm's debt contracts. This immediately raises the number of claimants in the bargaining process and may propel the firm into Chapter 11 proceedings. In addition, firms in default typically find it more difficult to arrange trade credit, and customers who depend upon a continuing relationship with the firm may abandon ship in the face of uncertainty about the firm's future operations.

[15] In our model, the likelihood that the covenant will bind is not dependent upon the stringency of the covenant. As long as this is true in an expanded model with states in which the covenant does not bind, then all of our results carry over exactly. A potentially interesting extension of the model would recognize that more stringent covenants increase the set of states in which the covenants bind. We discuss the effect of relaxing assumption (10) in Section 4.

D. *Renegotiation*

In addition to designing the payments and the covenant restrictions, we also permit the borrower and lender to design the contract so that renegotiation of contract terms is impossible, if they so choose. While the contracting literature has stressed the difficulties of enforcing a promise to refuse to renegotiate, certain contractual choices may ease or discourage renegotiation (see, for example, Hart and Moore (1988)).

For example, debt placed privately with a small number of large investors or a single bank loan may be much easier to renegotiate than public debt. When renegotiation requires costly monitoring—as it does in our model—a small number of heavily exposed investors will have greater incentives to bear the costs of staying informed. In addition, consensus is easier to obtain when the debt is closely held. As Hart and Moore (1989) have argued, widely dispersed debt may serve as a precommitment device that raises the costs of renegotiation. So our assumption that contracts might preclude renegotiation may be interpreted as the possibility of floating a public bond issue in preference to a private placement or bank loan.

However, the ability to renegotiate contracts in the light of new information may be quite valuable. In our setting, when renegotiation is possible, the borrower may seek to renegotiate the contract in period 2, before the audit establishes whether a breach has occurred.[16] For simplicity, we assume that the borrower makes a take-it-or-leave-it offer of a new contract to the lender, who may then decide to accept or reject. If the lender accepts, then the new contract is in force; if the lender rejects, then the original contract remains in force.[17] We exclude the possibility that the lender offers the borrower a side payment to forgo production.[18]

[16] Nothing changes if we assume that renegotiation follows, rather than precedes, the audit because the audit reveals only whether the covenant has been breached, not the borrower's type. However, it is important for our results that the firm must make its productive choice before renegotiation occurs. In particular, this assumption is the main reason that separating equilibria do not arise in the bargaining game. One may view this assumption as the polar case that best captures constraints on renegotiation in the real world, where decisionmaking is continuous and business opportunities are fleeting. Alternatively, this is the timing structure that would arise endogenously in a world where the firm's "type" refers to the quality of its management, and where bad managers are reluctant to identify themselves, because the personal losses of doing so exceed any feasible compensation by the firm's owners. Recent models of managerial behavior of this type include Diamond (1990), Harris and Raviv (1990), and Stulz (1990).

[17] This very simple bargaining game gives all bargaining power to the borrower, but more general bargaining games that result in a sharing of the bargaining surplus would yield identical results. The sole significance of the particular solution we choose is that it gives *some* bargaining power to the borrower.

[18] In the real world, a side payment would take the form of a prepackaged bankruptcy in which the firm's owners retain equity in the reorganized firm. We sometimes observe this type of agreement for firms in serious financial distress, that is, for firms that have already

To complete our description of contracts, we note that while the lender may have information about the borrower's type, contracts may not include terms that are conditioned explicitly on the borrower's type. Those factors affecting the state of the market are complicated, and no third-party contract enforcer can verify the firm's type at reasonable cost.

4. OPTIMAL DEBT CONTRACTS WITH AND WITHOUT RENEGOTIATION

If detailed, type-contingent contracts were feasible, but the lender could not liquidate selectively, then by (4) and (5) the first-best contract would direct the bad firm to invest all funds in safety activities and would direct the good firm to invest nothing in safety activities. Since these are not feasible, we now consider the optimal second-best contracts.

A. *Optimal Nonrenegotiable Contracts*

Consider a contract that includes a covenant of the form typical in debt contracts. The contract specifies a loan rate r^* and directs the borrower to keep safety activities above some minimum level, x^*. For this case, we assume that renegotiation outside of bankruptcy is impossible both before and after the audit. Thus, the covenant constraint is binding.

Before writing out the maximization problem, it is helpful to examine the incentive-compatible production decision for a borrower that has observed s_j. This can be written

$$(x_j^{\mathrm{g}}, x_j^{\mathrm{b}}) = \operatorname{argmax} \{p_{\mathrm{g}j} \max[R^{\mathrm{g}}(x_j^{\mathrm{g}}) - r, 0] + p_{\mathrm{b}j} \max[R^{\mathrm{b}}(x_j^{\mathrm{b}}) - r, 0]\} \quad (11)$$

given the resource constraint (3) and the covenant restriction

$$x_j^{\mathrm{b}} \geq x^*. \quad (12)$$

By condition (6), $R^{\mathrm{b}}(x) < r^*$, for any x and any feasible loan rate. So, as long as $p_{\mathrm{g}j} > 0$ and $R^{\mathrm{g}}(x_j^{\mathrm{g}}) > r^*$, a type j borrower will choose the minimum

---

defaulted. There are good reasons why we do not observe these types of agreements between institutional lenders and firms that are not in distress, that is, firms that can *choose* not to default (as in our model). First, manager's loss of control rents upon reorganization limits the feasibility of striking a mutually beneficial deal between the firm's lenders, owners, and managers. Second, side payments from an institutional lender to the firm will almost certainly be challenged by other claimants on the firm, like trade creditors. Third, rewards to the firm's owners or managers for bad performance have very undesirable incentive implications. In a world where the firm's type can be affected by effort, rewards for failure can reduce effort and increase the likelihood of failure.

level of the safety activity satisfying (12). The firm has no incentive to invest any more in the safety activity because while this would increase the value of the firm in the bad state, the lender, and not the firm would get the additional revenue, since the firm defaults in the bad state. Throughout, we assume that the optimal loan rate is low enough that a borrower has positive profit when the market turns out to be good. Thus, since $p_{gg} = 1 - \varepsilon > 0$, and $p_{gb} = \varepsilon > 0$, the borrower's optimal choices are

$$x_g^g = 1 - x^*, x_g^b = x^* \qquad \text{and} \qquad x_b^g = 1 - x^*, x_b^b = x^*, \tag{13}$$

recognizing that the resource constraint must bind, since the revenue functions are strictly increasing.[19]

Note that (13) holds for *any* positive values of p_{gj}. So, to keep the profit expressions uncluttered (and for this reason alone), we make the simplifying assumption that $\varepsilon \approx 0$; that is, the interim news about the ultimate state of the market is nearly perfect. Then the problem is to choose $\{r^*, x^*\}$ to maximize the borrower's expected profit subject to the lender's participation constraint and (13), or

$$\max_{\{r^*, x^*\}} p\{\max[R^g(x_g^g) - r, 0]\} + (1 - p)\{\max[R^b(x_b^b) - r, 0]\} \tag{14}$$

$$\text{s.t.} \quad p\{\min[r, R^g(x_g^g)]\} + (1 - p)\{\min[r, R^b(x_b^b)]\} \geq d, \tag{15}$$
$$\text{and (13).}$$

To solve for the optimal contract, substitute (15) and (13) into (14) to obtain the borrower's expected profit as a function of the covenant restriction,

$$pR^g(1 - x^*) + (1 - p)R^b(x^*) - d. \tag{16}$$

Maximizing this expression with respect to x^*, the first-order condition defines the optimal level of the covenant

[19] Assumptions (4) and (5) ensure that corner solutions for the borrower's investment choices are efficient. But suppose ε were large enough so that either (4) or (5) did not hold. Given condition (6), borrowers still would have no incentive to invest in safety activities and their equilibrium investment choices would still be given by (13). The only effect of relaxing (4) or (5) is that there is an interior solution for x^* over a larger range of values of p. If (6) did not hold, but (4) and (5) did hold, then there would be no agency problem; the optimal contract would be $\{r^* = d, x^* = 0\}$, the bad firm would invest only in safety activities, and the good firm would invest only in growth activities. If we were to relax (4)–(6) together, then the optimal contracting problem would become substantially more complicated, because a second agency problem—overinvestment in growth activities by the good firm—would arise.

$$-pR^{g}_{x}(1 - x^{*}) + (1 - p)R^{b}_{x}(x^{*}) = 0, \tag{17}$$

assuming an interior solution.[20]

Expression (17) makes the trade-offs clear. The first term captures the marginal cost of the covenant restriction: the good firm must invest in safety activities even though this yields no added revenue under good market conditions. The second term captures the marginal benefits of the covenant restriction: the bad firm is forced to invest in safety activities that it otherwise would not, thus increasing the period 3 value of the firm in the bad market. Since the value of the firm is increased by the use of the covenant, the borrower receives a lower loan rate than it otherwise would and so willingly accepts restrictive covenants in the debt contract.[21]

The optimal covenant will be less restrictive when agency problems are likely to be less severe. Totally differentiating expression (17) we find that

$$\frac{dx^{*}}{dp} = \frac{R^{g}_{x}(1 - x^{*}) + R^{b}_{x}(x^{*})}{pR^{g}_{xx}(1 - x^{*}) + (1 - p)R^{b}_{xx}(x^{*})} < 0, \tag{18}$$

since the numerator is positive by (2) and the denominator is negative by the concavity of $R^{j}(\cdot)$. As the probability of good market conditions increases, agency problems become less severe, so the optimal covenant is less stringent.

B. *Optimal Renegotiable Contracts*

Assume now that the contracting parties have adopted a contractual form that permits renegotiation of the contract, with r^{n} denoting the loan rate and x^{n} denoting the covenant restriction in the initial contract.[22] In this case, the borrower will consider the possibility of renegotiating the contract in period 2 when it makes its production decision in period 1.

We provide an intuitive discussion of the unique pure strategy equilibrium outcome.[23] Since the borrower knows whether the lender will be informed, the borrower's decisions in the informed and uninformed states can be analyzed separately.

Informed state. In the informed state, the good firm will seek renegotiation and propose a new contract $\{r' = L^{g}, x' = 0\}$, which the lender will

[20] The second-order condition is always satisfied, since the revenue functions are concave.

[21] See Iskandar and Emory (1990) for evidence that credit ratings respond positively to the inclusion of restrictive covenants.

[22] To ensure that r^{n} is high enough so there is an agency problem, we assume that $R^{b}(1) < (d - p\theta L^{g})/(1 - p\theta)$.

[23] See Appendix 2 for the formal derivation of the equilibrium production decisions and the bargaining outcome.

accept. The bad firm will not seek to renegotiate the contract. The borrower's investment choices are

$$x_{\mathrm{g}}^{\mathrm{g}} = 1, x_{\mathrm{g}}^{\mathrm{b}} = 0 \quad \text{and} \quad x_{\mathrm{b}}^{\mathrm{g}} = 1 - x^{n}, x_{\mathrm{b}}^{\mathrm{b}} = x^{n}. \tag{19}$$

Intuitively, the bad firm will never make a production decision in period 1 that will place it in breach of the original contract if it knows that the lender will be informed in period 2. Since the liquidation value of a bad firm is greater than any revenue it might produce in period 3 by (8), an informed lender will always refuse a new contract offer from a bad firm, because it would prefer to liquidate the firm. In turn, the bad borrower will never choose a level of safety activities lower than that specified in the original contract.[24]

Under assumption (9), the good firm will always request renegotiation. The good firm will invest everything in growth activities and nothing in safety activities in period 1 and will offer a new contract with no covenant restriction. Since the borrower makes a take-it-or-leave-it offer, it offers the lowest rate consistent with the lender's accepting the proposed new contract, L^{g}. We assume that if the lender is indifferent, it will accept the new offer.

At first glance, it might seem counterfactual that good firms seek renegotiation while bad firms do not. But it should be remembered that we are modeling renegotiations of "healthy" firms. Lummer and McConnell (1989) find that in their sample of 357 revised credit agreements, 259 were favorable revisions while only 22 were unfavorable (76 were mixed), and the favorable revisions were interpreted as good news by the market (significantly positive average excess returns).

Uninformed state. In the uninformed state, given assumption (10), there is no separating equilibrium and any pooling equilibrium involves the lender's liquidating rather than accepting any new contract offer.[25] Thus, neither type of firm will seek to renegotiate the contract, and the period 1 production choices are

$$x_{\mathrm{g}}^{\mathrm{g}} = 1 - x^{n}, x_{\mathrm{g}}^{\mathrm{b}} = x^{n}, \quad \text{and} \quad x_{\mathrm{b}}^{\mathrm{g}} = 1 - x^{n}, x_{\mathrm{b}}^{\mathrm{b}} = x^{n}. \tag{20}$$

The intuition behind this equilibrium is straightforward. In any proposed separating equilibrium, where the contract offer reveals the borrower's type, the lender would accept the good firm's offer and refuse the bad firm's offer. But if this is the lender's strategy, the bad borrower

[24] Clearly, no borrower could ever gain by proposing a more restrictive contract.

[25] Recall our earlier discussion of assumption (10). If the model were extended to include states of the world in which the covenant was not binding on the good firm, then assumption (10) would mean that if the borrower requests renegotiation, it is highly probable that it is bad. We discuss the implications of dropping assumption (10) following Proposition 1 below.

would always prefer to mimic the good borrower's offer. So, no separating equilibrium can exist.

No pooling equilibrium with both types of borrowers choosing a level of safety activities below x^n is possible. This follows since the lender would prefer to liquidate rather than accept any new contract with a lower covenant. Of course, neither type of borrower will place itself in breach of the initial contract just to be liquidated. So, both types satisfy the initial covenant and no renegotiation takes place.

We can now derive the optimal contract when renegotiation is permitted and state our first result:

PROPOSITION 1. *The covenant restriction in the renegotiable contract is more stringent than the covenant restriction in the nonrenegotiable contract.*

Proof. See Appendix 1.

The intuition behind Proposition 1 becomes clear when we compare the first-order condition for the optimal covenant in the nonrenegotiable contract (Eq. (17)) with the first-order condition for the optimal covenant in the renegotiable contract,

$$-p(1 - \theta)R_x^g(1 - x^n) + (1 - p)R_x^b(x^n) = 0. \tag{21}$$

For any covenant level x, the marginal cost of the covenant in the renegotiable contract (the first term of (21)) is smaller than the marginal cost of the covenant in the nonrenegotiable contract (the first term in (17)). This follows since in the renegotiable contract, the covenant imposes a costly constraint on the good firm only when the lender is uninformed and the contract cannot be renegotiated. The marginal benefit of the covenant in both contracts is equal. Thus, the optimal covenant in the renegotiable contract will be stricter than in the nonrenegotiable contract.[26]

Intuitively, renegotiation permits the covenant to be relaxed when the lender believes that it poses an inefficient constraint. Since this reduces the probability that the good borrower must invest in safety activities, the initial contract can impose stronger controls on the bad borrower. It can also be shown that $x^n - x^*$ is increasing in θ; as the effectiveness of the

[26] How would relaxing assumptions (4)–(6) affect the optimal renegotiable contract? One effect of relaxing assumptions (4) and (5) would be to widen the range of values of p for which an interior solution is optimal, as in the case of the nonrenegotiable contract. In this case, however, relaxing assumption (5) has an added effect. In the informative state, the good firm would still choose to invest only in growth activities given (6). But if ε were large enough so that the good firm should optimally make some investment in safety activities, renegotiation would lead to overinvestment in growth activities (relative to the first best). This would reduce the profitability of the renegotiable contract. However, this would not change x^n or affect any of our comparative statics results.

lender's monitoring apparatus improves, the good borrower must honor the very restrictive initial covenant less often, so the optimal covenant is more stringent. As with the nonrenegotiable contract, the first-order condition for the optimal covenant in the renegotiable contract (Eq. (21)) can be differentiated to show that $dx^n/dp < 0$.

What happens if we remove parametric restriction (10), which guarantees that no renegotiation occurs in the uninformed state? It is still true that only pooling equilibria are possible in the uninformed state. But if restriction (10) is violated for $x = x^*$, then there exists a pooling equilibrium in which both borrower types violate the initial covenant and propose a new contract that the lender will accept. Thus, the initial contract is unenforceable in the uninformed state. Under plausible parametric restrictions, the impossibility of enforcing the contract in the uninformed state means that the renegotiable contract yields negative expected profit for the lender. Given (8) and (9), it is easy to show that (10) will be satisfied for all values of p below some critical value, say p^*, and will be violated for all $p \geq p^*$.[27]

In the next section, we show that enforceable renegotiable contracts typically are more profitable than nonrenegotiable contracts when p is low and less profitable when p is high. So, if we remove restriction (10), there are two possibilities: (i) Over the range of values of p where the renegotiable contract is enforceable, that is for $p \leq p^*$, the renegotiable contract is more profitable than the nonrenegotiable contract for low values of p and is less profitable for high values of p, or (ii) over $p \leq p^*$, the renegotiable contract is more profitable than the nonrenegotiable contract for all values of p. In either case, our empirical predictions remain intact.

5. THE VALUE OF THE OPTION TO RENEGOTIATE: EMPIRICAL PREDICTIONS

In this section we present our main results, which relate the value of the option to renegotiate to the model's parameters. Our most interesting result concerns the effect of changes in p, the prior probability of a good market. At very low levels of p, an increase in p may increase the value of the option to renegotiate. Beyond some minimum level of p, further increases always reduce the value of this option. After presenting our results, we provide some intuition and interpretation. At the end of the section we discuss the possibility of using an alternative contract that forces the borrower to renegotiate—short-term debt that must be rolled

[27] The renegotiable contract will be infeasible if $d + k > pL^g + (1 - p)L^b$, that is, a lender would not invest funds and bear the costs of monitoring merely to have the option to liquidate the firm in states where the covenant binds. If restriction (8) is violated, (10) is automatically violated. Appendix 3 gives proofs of the results involving the relaxation of assumption (10).

over—and compare it with the contract in which renegotiation is always voluntary.

Letting π^n denote the borrower's expected profit under a renegotiable contract and π^* denote the borrower's expected profit under a nonrenegotiable contract, we use (16) and (A1.3) (in Appendix 1) to define the ex ante *value of the option to renegotiate:*

$$\begin{aligned}\pi^d \equiv \pi^n - \pi^* = p\{\theta R^g(1) + (1 - \theta)R^g(1 - x^n) - R^g(1 - x^*)\} \\ + (1 - p)\{R^b(x^n) - R^b(x^*)\} - k.\end{aligned} \tag{22}$$

Since $x^n > x^*$, we know that the second bracketed term is positive; that is, the revenue produced by the bad borrower is always higher under the renegotiable contract. The sign of the first bracketed term is ambiguous. When the lender is informed and the contract is renegotiated, the good borrower's revenue is higher under the renegotiable contract. But in the uninformed state, where the contract cannot be renegotiated, the good borrower must satisfy the more stringent covenant and revenue is lower. Thus,

$$\frac{d\pi^d}{d\theta} = p\{R^g(1) - R^g(1 - x^n)\} > 0, \tag{23}$$

where (23) is derived using the Envelope Theorem and is positive since $R^g(\cdot)$ is an increasing function. The higher the probability that the lender will learn the borrower's type, the more likely it is that the good borrower will not have to satisfy the more restrictive covenant x^n and the more valuable the option to renegotiate. In addition, it is obvious that the value of the option to renegotiate is lower when monitoring is more costly. Summarizing, we have:

PROPOSITION 2. *The value of the option to renegotiate is increasing in the probability of the lender's being informed, θ, and decreasing in the costs of monitoring, k.*

Proof. See Appendix 1.

To determine the effect of the firm's ex ante creditworthiness on the value of the option to renegotiate, we differentiate (22) with respect to p and use the Envelope Theorem to obtain

$$\begin{aligned}\frac{d\pi^d}{dp} &= \{\theta R^g(1) + (1 - \theta)R^g(1 - x^n) - R^g(1 - x^*)\} - \{R^b(x^n) - R^b(x^*)\} \\ &= \{\theta[R^g(1) - R^g(1 - x^n)] - [R^g(1 - x^*) - R^g(1 - x^n)]\} \\ &\quad - \{R^b(x^n) - R^b(x^*)\}.\end{aligned} \tag{24}$$

The first line of expression (24) shows that the sign of $d\pi^d/dp$ depends upon the size of the good borrower's expected revenue gain—taking into account the probability that it will be forced to honor a more restrictive covenant with probability $1 - \theta$—compared with the size of the revenue gain from tighter control of the bad borrower. The derivative $d\pi^d/dp$ will be negative when the good borrower's expected revenue gain is small and bad borrower's revenue gain is large.

We are primarily concerned about the behavior of π^d over the region where both contracts have interior solutions for the optimal covenant. But evaluating expression (24) at extremely high and low levels of p, where contracts yield corner solutions, provides some useful information.

PROPOSITION 3. *If for some value of p, $0 < x^n(p) < 1$ and $0 < x^*(p) < 1$, then there exist p^a, p^b, p^c, p^d, with $p^a < p^b < p^c < p^d$, for which the following hold:*

(i) $d\pi^d/dp > 0, \quad 0 < p < p^a,$
(ii) $d\pi^d/dp < 0, \quad p^c < p < p^d,$
(iii) $d\pi^d/dp = 0, \quad p^d \le p \le 1,$ *and*
(iv) $d^2\pi^d/dp^2 \le 0, \quad p \le p^b.$

Proof. See Appendix 1.

Proposition 3 states that $d\pi^d/dp$ is positive and nonincreasing for very low values of p and that it is negative for very high values of p—until p is high enough that both contracts prescribe no covenant at all. When both contracts prescribe no covenant, there is nothing to renegotiate, and non-renegotiable contracts dominate. As shown in Fig. 2, $[p^b, p^c]$ is the interval on which both contracts have interior solutions for the optimal covenant. For p outside this interval, either one or both contracts have corner solutions. It should be noted that without any further restrictions on the functional forms, part (ii) of Proposition 3 guarantees that there exists some critical value of p on the interval $[p^b, p^c]$, beyond which the value of the option to renegotiate is strictly decreasing (until both contracts prescribe no covenant).

Proposition 4 provides a plausible sufficient condition for the value of the option to renegotiate to fall continuously as p increases above some critical value.

PROPOSITION 4. *If π^d is quasiconcave for all values of p where both $0 < x^n(p) < 1$ and $0 < x^*(p) < 1$, then there exists some $p' \in [p^a, p^c]$ such that*

(i) $d\pi^d/dp > 0, \quad 0 < p < p',$
(ii) $d\pi^d/dp < 0, \quad p' < p < p^d,$ *and*
(iii) $d\pi^d/dp = 0, \quad p^d \le p \le 1.$

Proof. See Appendix 1.

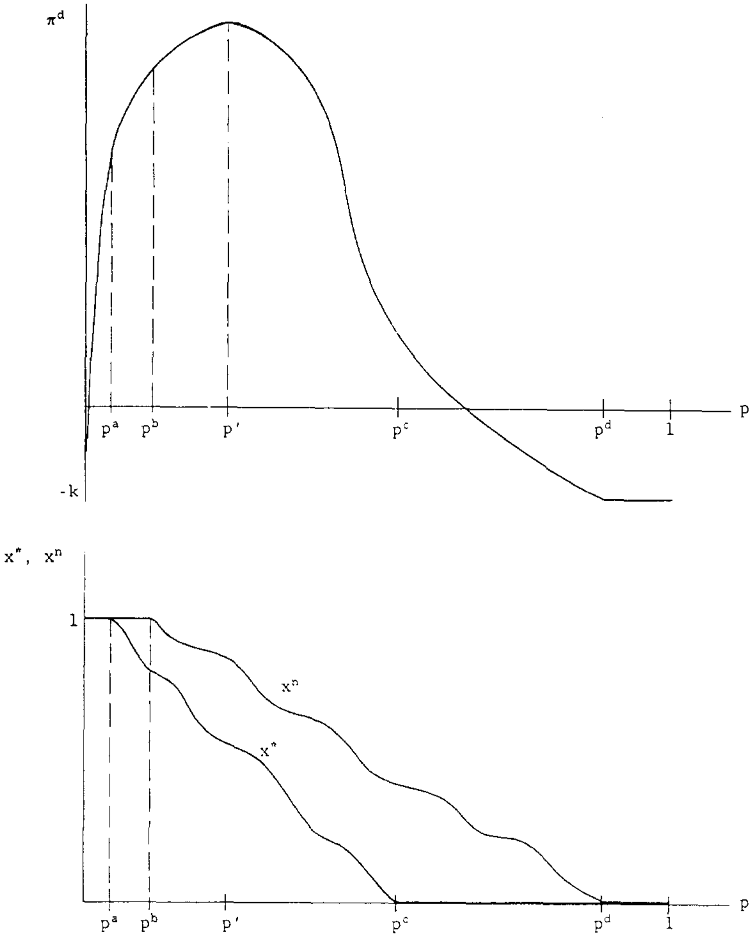

FIG. 2. π^d is increasing for all $p < p'$ and decreasing for all $p > p'$.

Figure 2 illustrates a case in which π^d first increases and then decreases as p increases over the interval $[p^b, p^c]$ where both contracts have interior solutions. Note that it is possible for π^d to be decreasing in p over the entire interval with interior solutions; in particular, this will be true if $p' < p^b$.

The conditions of Proposition 4 are satisfied for many reasonable revenue functions because the behavior of π^d is driven primarily by the concavity of the revenue functions.[28] Compared with the nonrenegotiable

[28] Examples with negative exponential revenue functions and with quadratic revenue are shown in Appendix 4. In both cases, π^d is quasiconcave over the region where there are interior solutions.

contract, the renegotiable contract is beneficial because it permits a stricter covenant to be imposed on the bad firm—this gain is measured by $R^b(x^n) - R^b(x^*)$—and because it allows the good firm to renegotiate a less restrictive covenant when the lender is informed—this gain is measured by $R^g(1) - R^g(1 - x^n)$. The cost of the renegotiable contract is that the good firm is forced to honor the more stringent covenant when the lender is uninformed—this loss is measured by $R^g(1 - x^*) - R^g(1 - x^n)$. As p increases, there are two opposing effects: First, since the borrower is more likely to be good, the probability of beneficial renegotiation increases, which tends to make the option to renegotiate more valuable. Second, since agency problems are less likely, the value of more stringent control over the bad borrower decreases, which tends to make the option to renegotiate less valuable.

The concavity of the revenue functions implies the second effect dominates, for a given difference $x^n - x^*$.[29] As p increases, both x^n and x^* decrease, implying that optimal covenants become less stringent. So, when p is large, $R^b(x^n) - R^b(x^*)$ is large, implying that the marginal gains from greater stringency fall rapidly in p. Further, for large p, *both* the gain from renegotiating the stringent contract—measured by $[R^g(1) - R^g(1 - x^n)]$—and the cost of excessive stringency when the lender is uninformed—measured by $[R^g(1 - x^*) - R^g(1 - x^n)]$—are small. Thus, the net benefits for the good firm become negligible when p is large. As indicated by the second equality in (24), this means that the value of the option to renegotiate will be decreasing for large enough p.

In essence, as the probability of good market conditions increases, the control of agency problems becomes less important. When good market conditions are unlikely and agency problems are severe, optimal covenants are very stringent. So, the option to relax these covenants selectively is quite valuable. When good market conditions are very likely, optimal covenant restrictions are less stringent, and the benefits of being able to relax covenants selectively grow smaller.

Our model's predictions are consistent with empirical findings concerning a firm's choice between publicly placed debt, on the one hand, and privately placed debt and bank loans, which are easier to renegotiate, on the other. Our model predicts that a particular firm's debt contracts will have more stringent covenants when the debt is closely held, thus providing theoretical support for a conjecture of Smith and Warner (1979). Our model also predicts that the most creditworthy firms will float public debt and that less creditworthy firms will take bank loans or place their debt privately. It is the less creditworthy firms that benefit most from debt contracts with stringent covenants and, in turn, the option to reduce the costs of stringent covenants by renegotiation. This result is consistent

[29] In general, $x^n - x^*$ does not remain constant as p increases.

with the empirical research of Blackwell and Kidwell (1988) who find that speculative-grade issuers would have paid substantially more to sell their debt publicly rather than privately. They thus conclude that there is a positive relationship between a firm's agency problems and the benefits of issuing privately.

Restrictive Covenants and Debt Maturity

Implicit in our model of renegotiable debt is that it is optimal to control the borrower through contractual provisions that trigger renegotiation only when the borrower *chooses* to violate the initial covenant. Knowing that they will be bargaining from a position of weakness, the bad borrower in the informed state and both borrowers in the uninformed state choose to honor the initial contract. We have not considered the possibility that the initial contract might *force* the borrower to renegotiate. This would be the case if the borrower and lender signed a short-term debt contract requiring payments in period 2. Then, since no revenues can be produced in period 2, the firm would avoid liquidation only if it were able to roll over the loan.[30]

Given restrictions (8)–(10), only the good firm in the informed state could roll over its loan, and the lender would liquidate in all other states. If we permit the lender to make up-front payments to the borrower—necessary for zero expected profit for the lender—then the short-term contract strictly dominates the long-term contract with covenants: Let π^s denote the total expected surplus to be divided between the borrower and lender under the short-term contract. Since the expected profit of the lender is zero under the renegotiable contract, π^n represents the total expected surplus of the renegotiable contract. Then using Eq. (A1.3) of Appendix 1 we have

$$\pi^s - \pi^n = -p(1 - \theta)[R^g(1 - x^n) - L^g] + (1 - p)[L^b - R^b(x^n)] > 0.\text{[31]} \tag{25}$$

Intuitively, the first term reflects the loss in efficiency from using short-term debt—the inefficient liquidation of the good borrower when the lender is uninformed. The second term represents the gain in efficiency—the efficient liquidation of the bad firm in all states. Given restriction (10), the expected gain outweighs the expected loss.

[30] We thank an anonymous referee for these insights.

[31] To show $\pi^s - \pi^n > 0$, observe that the second term in (25) is positive by (8). Therefore, if $R^g(1 - x^n) - L^g < 0$ then $\pi^s - \pi^n > 0$. To prove $\pi^s - \pi^n > 0$ when $R^g(1 - x^n) - L^g > 0$, rewrite (25) as $\pi^s - \pi^n = \{[pL^g + (1 - p)L^b] - [pR^g(1 - x^n) + (1 - p)R^b(x^n)]\} + p\theta[R^g(1 - x^n) - L^g]$. By (10), the term in braces is positive. Therefore, $R^g(1 - x^n) - L^g > 0$ implies $\pi^s - \pi^n > 0$.

It is worth noting that if the cost of the monitoring apparatus, k, is high enough, and the ex ante creditworthiness of the borrower, p, is low enough, then short-term debt will dominate the nonrenegotiable contract, which will dominate the renegotiable contract (i.e., $\pi^s > \pi^* > \pi^n$). With p low, liquidation is attractive, but if the monitoring apparatus is too costly, then monitoring by forced liquidation may be optimal. Thus, short-term debt may dominate both long-term contracts.

A full analysis of the trade-offs between the use of short-term debt with forced renegotiation and debt with restrictive covenants is beyond the scope of the current paper. Here, we suggest merely three variations of the basic model in which forced renegotiation need not dominate.

(i) Consider again the existence of states of the world where the covenant does not bind the good firm. Then short-term debt may give the lender too much power in these states, allowing it to extract higher rents from the firm and causing distortions, as in Rajan (1991). Long-term, renegotiable debt would prevent this.

(ii) We have assumed that the informed lender is the sole creditor. Consider, however, a model in which the informed lender holds only a share (δ) of the firm's debt, and assume that the informed lender and other creditors have equal priority. Assume further that restrictions (8) and (9) hold, but that $pR^g(1 - x^*) + (1 - p)R^b(x^*) > pL^g + (1 - p)L^b$; that is, (10) does not hold and liquidation is *not* efficient in the uninformed state. Under these assumptions, inequality (25) is reversed, and the renegotiable contract with covenants dominates short-term debt *if it is enforceable*. But in Section 4 we showed that renegotiable contracts are infeasible in this case because the covenant is unenforceable in the uninformed state. We can reestablish the enforceability of our contract with restrictive covenants by giving the informed lender a higher priority claim than other uninformed creditors. Letting δ^d denote the informed lender's share of the firm's assets in liquidation, a sufficient condition for reestablishing the enforceability of the covenant restriction would be

$$\delta pR^g(1 - x^*) + \delta^d(1 - p)R^b(x^*) < \delta^d[pL^g + (1 - p)L^b],$$

so the informed lender's *private* share of the firm's assets in liquidation is high enough that it would prefer to liquidate the firm if it defaulted on the covenant in the uninformed state. Thus, both firm types will honor the covenant (as in our previous analysis) and renegotiable contracts with restrictive covenants dominate short-term debt with forced renegotiation.[32]

[32] Introducing uninformed creditors is *not* a trivial extension of the model, as this explanation might suggest. A full analysis would require careful consideration of the potential conflicts of interest between the informed lender and uninformed creditors.

(iii) Consider another extension of the basic model, where restriction (10) again holds but the firm's owner/manager has per-period control rents (C), that are received as long as the firm is not liquidated and that are nontransferrable (as in Diamond (1990), for example). Note that the introduction of control rents has no effect on the value of the option to renegotiate—expression (22)—because the firm never voluntarily defaults in period 2. It does, however, affect the relative efficiency of short-term contracts with forced renegotiation and contracts with voluntary renegotiation. In particular, since under the short-term contract, the lender liquidates the firm in period 2 unless the lender knows that the firm is a good type, the lost control rents are $[p(1 - \theta) + (1 - p)]C$. If these are large enough, which is most likely in privately or closely held firms, either of the contracts with restrictive covenants might dominate short-term debt with forced renegotiation.

6. CONCLUSION AND DIRECTIONS FOR FURTHER RESEARCH

In this paper we have explored the connections between the stringency of covenant restrictions and the ease of renegotiating debt contracts. Restrictive covenants are written into debt contracts to control agency problems, but these restrictions are not costless because they restrict the borrower's flexibility to make efficient investment decisions. If creditors' costs of distinguishing legitimate from illegitimate requests to renegotiate contractual covenants are not too high, then the option to renegotiate contracts is valuable. By writing very restrictive covenants, which can be selectively renegotiated, agency problems are reduced without unduly constricting the borrower's ability to make valuable investments.

Our main results concern the impact of the firm's creditworthiness on the value of the renegotiation option. We find that when the firm's creditworthiness is very low, a rise in creditworthiness will increase the value of the option to renegotiate. Beyond some minimum level, however, further increases in the firm's creditworthiness reduce the value of the option to renegotiate. In addition, we demonstrate that covenant restrictions become less severe as creditworthiness improves, whether or not the contract can be renegotiated, and that the value of the option to renegotiate increases as the creditor's monitoring technology improves. These predictions are consistent with the anecdotal and empirical evidence concerning factors affecting a firm's choice between publicly placed debt, on the one hand, and privately placed debt and bank loans on the other.

While we feel our results are illuminating, we do not claim to have unearthed a definitive explanation for the firm's choice between different types of debt. Our model is silent about the relationship between the complexity of the covenants in the bond indenture and the potential size of the secondary market for the bond. Complex, tailor-made covenants, while they may reduce a firm's borrowing costs by more effectively controlling agency problems, should also reduce the liquidity of the firm's debt. Exploring this connection should yield more insight into the factors governing the firm's choice.

A second interesting departure would be an analysis of optimal contract design when the firm borrows through a mixture of closely held and publicly held debt. A few papers, notably, Brown *et al.* (1992), Bulow and Shoven (1978), Gertner and Scharfstein (1991), and White (1980), have examined debt renegotiations in which the firm has both private and public debt. Although both Gertner and Scharfstein and Bulow and Shoven discuss the relationship between the types of inefficiencies that arise in financial distress and the firm's initial debt structure, none of the papers has formally examined optimal contract design in the face of the types of debtholder conflicts that arise when the firm has both kinds of debt. This seems like an especially fruitful avenue for further research.

APPENDIX 1: PROOFS OF PROPOSITIONS

A. *Proof of Proposition* 1

The optimal contract solves

$$\max_{\{r^n, x^n\}} p\{\theta[R^g(1) - L^g] + (1 - \theta)[R^g(1 - x^n) - r^n]\} \qquad \text{(A1.1)}$$

$$\text{s.t. } p\{\theta L^g + (1 - \theta)r^n\} + (1 - p)R^b(x^n) - k \geq d. \qquad \text{(A1.2)}$$

For brevity, we have evaluated the min and max operators using conditions (6) and (7) in the text, as in our earlier derivation. The expression for the borrower's expected profit (A1.1) and the lender's participation constraint (A1.2) already incorporate the results of our discussion of the bargaining equilibrium (summarized in (19) and (20) in the text): With probability θ, the good borrower chooses the efficient level of safety activities and proposes a new contract to ratify its decision, which is accepted by the lender. With probability $1 - \theta$ the good borrower honors the initial contract and no renegotiations occur. The bad borrower always honors the initial contract and never seeks renegotiation.

Substituting (A1.2) into (A1.1), the borrower's expected profit can be written as a function of the covenant restriction:

$$p[\theta R^g(1) + (1 - \theta)R^g(1 - x^n)] + (1 - p)R^b(x^n) - k - d. \quad \text{(A1.3)}$$

Maximizing this expression with respect to x^n we obtain the first-order condition for the optimal covenant

$$-p(1-\theta)R_x^{g}(1-x^n)+(1-p)R_x^{b}(x^n)=0, \tag{A1.4}$$

assuming we have an interior solution.

Comparing expressions (A1.4) and (17) in the text, the corresponding first-order condition for the nonrenegotiable contract, it is immediate that $x^n > x^*$, since the revenue functions are increasing and concave, so that the optimal covenant in the renegotiable contract is more restrictive than the covenant in the nonrenegotiable contract. Q.E.D.

B. *Proof of Proposition* 2

Differentiating (22) with respect to θ and using the Envelope Theorem yields

$$\frac{d\pi^d}{d\theta}=p\{R^{g}(1)-R^{g}(1-x^n)\},$$

which is positive since $R^{g}(\cdot)$ is increasing. Differentiating (22) with respect to k yields

$$\frac{d\pi^d}{dk}=-1. \qquad \text{Q.E.D.}$$

C. *Proof of Proposition* 3

Since the conditions of the Theorem of the Maximum are satisfied, $\pi^p(x^n(p), p)$ is a continuous function of p. Define p^{b} as $\inf\{p\colon x^n<1$ and $x^*<1\}$. Similarly, define p^{c} as $\sup\{p\colon x^n>0$ and $x^*>0\}$. Then for $p\in(p^{b}, p^{c})$, x^n and x^* are interior solutions. Finally, define p^{a} as $\inf\{p\colon x^*<1\}$ and p^{d} as $\sup\{p\colon x^n>0\}$. Using (24) in the text and simplifying yields

$$\frac{d\pi^d(p)}{dp}=-(1-\theta)\{R^{g}(1)-R^{g}(1-x^n)\}-\{R^{b}(x^n)-R^{b}(0)\} \quad \text{for } p\geq p^{c}. \tag{A1.5}$$

For $p^{c}\leq p<p^{d}$, since $R^{j}(\cdot)$ is strictly increasing, both the first and second bracketed terms are positive. So the total expression is negative, that is, $d\pi^d(p)/dp<0$. Thus, at levels of p high enough so that only the nonrenegotiable contract prescribes no covenant, the value of the option to renegotiate is decreasing in p.

For all values of p high enough that both contracts prescribe no covenants, that is, for $p^{d} \leq p \leq 1$, it is obvious that no renegotiation would ever occur, since both firms are unconstrained. So by (22) in the text, $\pi^{d} = -k$, and so $d\pi^{d}(p)/dp = 0$.

For all $p \leq p^{a}$, $x^{n} = x^{*} = 1$, and expression (22) in the text implies

$$\pi^{d} = p\theta\{R^{g}(1) - R^{g}(0)\} - k, \tag{A1.6}$$

which is clearly increasing and (weakly) concave in p. So over some initial region where both contracts have the most stringent covenants possible, $d\pi^{d}(p)/dp$ is positive and nonincreasing.

Finally, for all p such that $p^{a} < p \leq p^{b}$, $x^{n} = 1$ and x^{*} has an interior solution. So, by (24),

$$\frac{d\pi^{d}(p)}{dp} = \theta R^{g}(1) + (1 - \theta)R^{g}(0) - R^{g}(1 - x^{*}) - \{R^{b}(1) - R^{b}(x^{*})\}. \tag{A1.7}$$

Differentiating this expression with respect to p yields

$$\frac{d^{2}\pi^{d}(p)}{dp^{2}} = \{R_{x}^{g}(1 - x^{*}) + R_{x}^{b}(x^{*})\}\left\{\frac{dx^{*}}{dp}\right\}, \tag{A1.8}$$

which is negative, since assumption (2) implies that the first bracketed term is positive and since dx^{*}/dp is negative. Thus, π^{d} is concave for all $p \leq p^{b}$. Q.E.D.

C. *Proof of Proposition* 4

By Proposition 3, $d\pi^{d}/dp$ is positive at p^{a} and negative at p^{c}. Then quasiconcavity of π^{d} on the interval $[p^{b}, p^{c}]$ guarantees that $d\pi^{d}/dp$ cannot equal zero at more than one p on this interval. Q.E.D.

APPENDIX 2: DERIVATION OF THE UNIQUE EQUILIBRIUM OUTCOME

Let $\pi^{j}(r, x)$ be the expected profit of a firm that observes signal j at contract (r, x). Then $\pi^{j}(r, x) = p_{gj} \max[R^{g}(1 - x) - r, 0] + p_{bj} \max[R^{b}(x) - r, 0]$. So the bad firm's expected profit at contract (r, x) is $\pi^{b}(r, x) = \varepsilon \max[R^{g}(1 - x) - r, 0] + (1 - \varepsilon) \max[R^{b}(x) - r, 0]$ and the good firm's expected profit at contract (r, x) is $\pi^{g}(r, x) = (1 - \varepsilon) \max[R^{g}(1 - x) - r, 0] + \varepsilon \max[R^{b}(x) - r, 0]$. We analyze the equilibrium

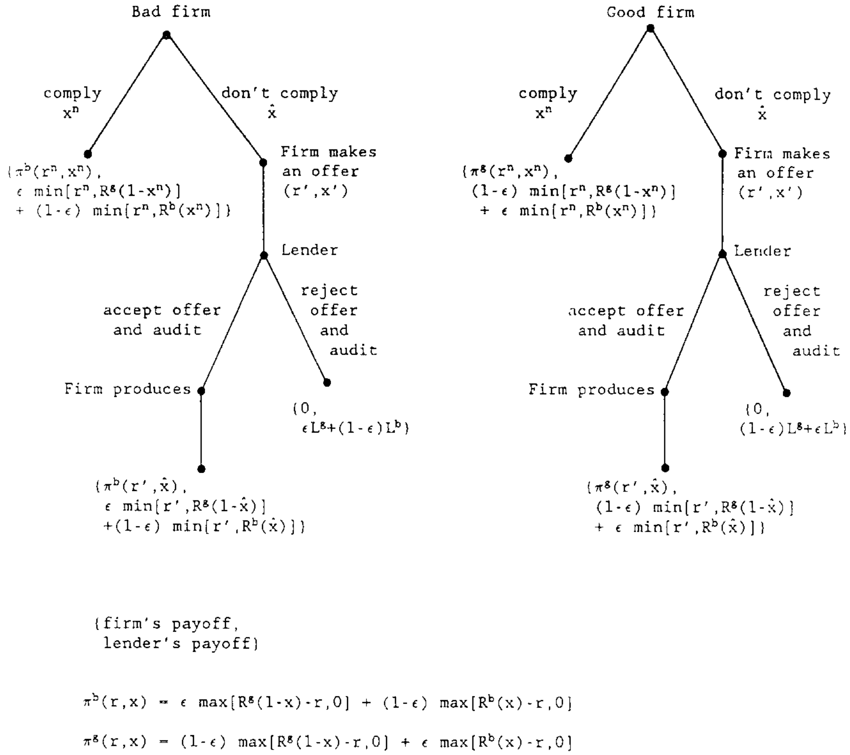

FIG. 3. Fully informed lender.

for ε sufficiently small. (We assume that if the lender is indifferent between rejecting or accepting an offer, it will accept.)

Lender Fully Informed

Refer to Fig. 3: $\hat{x}$ is the firm's choice and (r', x') is the firm's offer.

Consider the bad firm and use backward induction. At the lender's node, it would rather liquidate the firm than accept the offer. This is because by liquidating the firm the lender receives $\varepsilon L^g + (1 - \varepsilon)L^b \approx L^b$ for small ε, and by accepting the offer, the lender receives $\varepsilon \min[r', R^g(1 - \hat{x})] + (1 - \varepsilon) \min[r', R^b(\hat{x})]$, which for small ε is less than L^b. (To see this, note that for small ε, $\varepsilon \min[r', R^g(1 - \hat{x})] + (1 - \varepsilon) \min[r', R^b(\hat{x})] \approx \min[r', R^b(\hat{x})] < R^b(1) < L^b$ by Eq. (8) in the text.) So the lender will reject the offer.

Given that the lender will reject the offer, will the firm make an offer or comply with the original contract? If the firm complies, its profit $= \pi^b(r^n, x^n)$. If the firm does not comply, then its profit is 0, since the firm liquidates. Therefore, the firm will comply, since $\pi^b(r^n, x^n) > 0$. So there is a unique sequential equilibrium involving the following strategies: {bad firm complies, lender rejects offer}.

Consider the good firm. At the lender's node, if it rejects the offer, it receives $(1 - \varepsilon)L^g + \varepsilon L^b$, and if it accepts the offer, it receives $(1 - \varepsilon) \min[r', R^g(1 - \hat{x})] + \varepsilon \min[r', R^b(\hat{x})]$. Therefore, the lender accepts the offer $\Leftrightarrow (1 - \varepsilon) \min[r', R^g(1 - \hat{x})] + \varepsilon \min[r', R^b(\hat{x})] \geq (1 - \varepsilon)L^g + \varepsilon L^b$.

Suppose the lender accepts the offer; then what will the firm do? The firm will comply if $\pi^g(r^n, x^n) > \pi^g(r', \hat{x})$. So if $\pi^g(r^n, x^n) < \pi^g(r', \hat{x})$ and $(1 - \varepsilon) \min[r', R^g(1 - \hat{x})] + \varepsilon \min[r', R^b(\hat{x})] \geq (1 - \varepsilon)L^g + \varepsilon L^b$, then the firm makes the offer (r', x') and the lender accepts. The firm would receive $\pi^g(r', \hat{x})$ and lender would receive $(1 - \varepsilon) \min[r', R^g(1 - \hat{x})] + \varepsilon \min[r', R^b(\hat{x})]$. Given this, what would the firm offer? The best the firm can do is $x' = \hat{x} = 0$, $r' = L^g + [\varepsilon/(1 - \varepsilon)][L^b - R^b(0)]$. Then the lender's profit $= (1 - \varepsilon) \min[r', R^g(1)] + \varepsilon \min[r', R^b(0)] = (1 - \varepsilon)r' + \varepsilon R^b(0) = (1 - \varepsilon)\{L^g + [\varepsilon/(1 - \varepsilon)] [L^b - R^b(0)]\} + \varepsilon R^b(0) = (1 - \varepsilon)L^g + \varepsilon L^b$, so the contract is consistent with the lender's accepting.

And given that this offer is accepted by the lender, the firm would indeed want to make the offer. (To see this, note that if the firm makes the offer and it is accepted, the firm receives $\pi^g(r', \hat{x}) = R^g(1 - x') - r' = R^g(1) - L^g - [\varepsilon/(1 - \varepsilon)][L^b - R^b(0)] \approx R^g(1) - L^g$ for small ε. If the firm does not make an offer, it receives $\pi^g(r^n, x^n) = R^g(1 - x^n) - r^n = R^g(1 - x^n) - [d + k - (1 - p)R^b(x^n) - p\theta L^g]/p(1 - \theta)$ (since by Eq. (A1.2) in Appendix 1, zero profit for the lender implies $r^n = [d + k - (1 - p)R^b(x^n) - p\theta L^g]/p(1 - \theta)$.) As long as the ex ante expected return from liquidating the firm is less than the risk-free rate d, that is, as long as always liquidating is a negative net present value project (note that this does *not* mean that liquidating is better then not renegotiating) then the firm is better off making the offer. Proof: If always liquidating is a negative NPV project, then $pL^g + (1 - p)L^b - d < 0 \Rightarrow pL^g + (1 - p)R^b(x^n) - d - k < 0$ (since $R^b(x^n) < L^b$ by assumption (8)). But this implies, $p(1 - \theta)[R^g(1) - R^g(1 - x^n)] > pL^g + (1 - p)R^b(x^n) - d - k \Rightarrow R^g(1) - L^g > R^g(1 - x^n) - [d + k - (1 - p)R^b(x^n) - p\theta L^g]/p(1 - \theta) \Rightarrow \pi^g(r', \hat{x}) > \pi^g(r^n, x^n)$.

Therefore, one Nash equilibrium involves the following strategies: {firm chooses $\hat{x} = 0$ and offers $(r', x') = (L^g + [\varepsilon/(1 - \varepsilon)][L^b - R^b(0)], 0)$, lender accepts offer}.

Suppose now that the lender rejects the offer, that is, $(1 - \varepsilon) \min[r', R^g(1 - \hat{x})] + \varepsilon \min[r', R^b(\hat{x})] < (1 - \varepsilon)L^g + \varepsilon L^b$. Then what would the firm do? If the lender rejects the offer, then the firm gets 0 if it does not comply and $\pi(r^n, x^n)$ if it does comply. Therefore, the firm complies since $\pi(r^n, x^n) > 0$. So, if the lender rejects the offer the firm will comply. Hence, another Nash equilibrium involves the following strategies: {firm complies, lender rejects offer}. However, this Nash equilibrium is not a se-

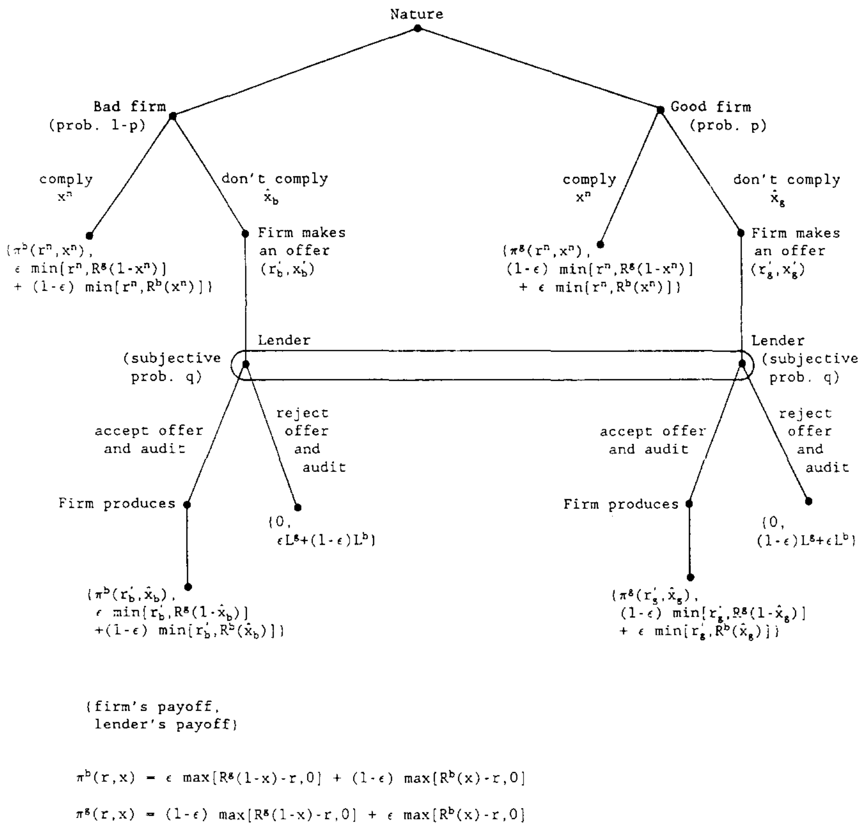

FIG. 4. Uninformed lender.

quential equilibrium because once the lender's node is reached, it is optimal for the lender to accept the offer.

Thus, there is a unique sequential equilibrium involving the strategies: {good firm chooses $\hat{x} = 0$ and offers $(r', x') = (L^g + [\varepsilon/(1 - \varepsilon)][L^b - R^b(0)], 0)$, lender accepts offer}.

Lender Uninformed

Refer to Fig. 4: $\hat{x}_b$ is the bad firm's choice and $\hat{x}_g$ is the good firm's choice.

First check the existence of any pooling equilibrium: Assume that the firms make the same offer (r', x').

Let q be the probability the lender assigns to the firm's being good if its information set is reached. $1 - q$ is the probability the lender assigns to the firm's being bad if its information set is reached. Will the lender accept or reject the offer (r', x')? The lender accepts if its expected profit from accepting $\geq$ expected profit from rejecting. The expected profit from accepting is $(1 - q)\{\varepsilon \min[r', R^g(1 - \hat{x})] + (1 - \varepsilon) \min[r', R^b(\hat{x})]\} + q\{(1 - \varepsilon) \min[r', R^g(1 - \hat{x})] + \varepsilon \min[r', R^b(\hat{x})]\}$. So, for small ε, the maximum profit the lender can receive if it accepts is $(1 - q)R^b(\hat{x}_b) + qR^g(1 - \hat{x}_g)$. (There are four cases: If $r' < R^b(\hat{x}_b)$ and $r' < R^g(1 - \hat{x}_g)$, then the lender's profit if it accepts is $(1 - q)\{\varepsilon r' + (1 - \varepsilon)r'\} + q\{(1 - \varepsilon)r' + \varepsilon r'\} = r' < (1 - q)R^b(\hat{x}_b) + qR^g(1 - \hat{x}_g)$. If $r' < R^b(\hat{x}_b)$ and $r' > R^g(1 - \hat{x}_g)$, then the profit from accepting is $(1 - q)\{\varepsilon R^g(1 - \hat{x}_g) + (1 - \varepsilon)r'\} + q\{(1 - \varepsilon)R^g(1 - \hat{x}_g) + \varepsilon r'\} < (1 - q)R^b(\hat{x}_b) + q R^g(1 - \hat{x}_g)$ for small ε. If $r' > R^b(\hat{x}_b)$ and $r' < R^g(1 - \hat{x}_g)$, then the profit from accepting is $(1 - q)\{\varepsilon r' + (1 - \varepsilon)R^b(\hat{x}_b)\} + q\{(1 - \varepsilon)r' + \varepsilon R^b(\hat{x}_b)\} < (1 - q)R^b(\hat{x}_b) + q R^g(1 - \hat{x}_g)$ for small ε. If $r' > R^b(\hat{x}_b)$ and $r' > R^g(1 - \hat{x}_g)$ then the profit from accepting is $(1 - q)\{\varepsilon R^g(1 - \hat{x}_g) + (1 - \varepsilon)R^b(\hat{x}_b)\} + q\{(1 - \varepsilon)R^g(1 - \hat{x}_g) + \varepsilon R^b(\hat{x}_b)\} \approx (1 - q)R^b(\hat{x}_b) + q R^g(1 - \hat{x}_g)$ for small ε.)

If the lender rejects the offer, its expected profit is $(1 - q)\{\varepsilon L^g + (1 - \varepsilon)L^b\} + q\{(1 - \varepsilon)L^g + \varepsilon L^b\}$, which for small $\varepsilon \approx (1 - q)L^b + qL^g$.

So if $(1 - q)R^b(\hat{x}_b) + q R^g(1 - \hat{x}_g) < (1 - q)L^b + qL^g$, the lender will reject the offer.

Suppose the lender's belief q is such that it rejects the offer. Then, what is the bad firm's best response to this? Given that the lender rejects the offer, the bad firm receives profit of 0 if it does not comply, and profit is $\pi^b(r^n, x^n)$ if it does comply. Therefore, the bad firm will comply, since $\pi^b(r^n, x^n) > 0$.

What is the good firm's best response to the lender's rejecting? Given that the lender rejects the offer, the good firm receives profit of 0 if it does not comply and profit is $\pi^g(r^n, x^n)$ if it does comply. Therefore, the good firm will comply since $\pi^g(r^n, x^n) > 0$.

Note that if both firms comply, then the lender's information set is never reached. Hence, there is a continuum of Nash equilibria: {good firm comply, bad firm comply, lender reject offer} supported by any belief q such that $(1 - q)R^b(x^n) + q R^g(1 - x^n) < (1 - q)L^b + qL^g$. These Nash equilibria are sequential equilibria. The reason is that, since the information set of the lender is not reached, we can specify any action on the part of the lender as long as it is supported by some belief. (If the lender's information set were reached in some Nash equilibrium, then we would have to specify the lender's belief using Bayes' Rule. But since the information set is not reached, we are free in specifying beliefs.) We can say there is a unique sequential equilibrium *outcome*—comply.

Now, check the existence of any separating equilibrium. Assume that the bad firm makes the offer (r_b', x_b') and the good firm makes the offer (r_g', x_g').

If the lender saw (r_g', x_g'), it would place a probability of 1 that the firm is good and would accept the offer as long as $(1 - \varepsilon) \min[r_g', R^g(1 - x_g')] + \varepsilon \min[r_g', R^b(x_g')] \geq (1 - \varepsilon)L^g + \varepsilon L^b$ (we assume that if there is a tie, the lender will accept the offer). The best action and offer the good firm could make if the offer is accepted and ε is small is $\hat{x}_g = 0$, $x'_g = 0$, $r'_g = L^g + [\varepsilon/(1-\varepsilon)][L^b - R^b(0)]$, since this would maximize the good firm's profit $\approx R^g(1) - L^g$ for small ε.

If the lender saw (r'_b, x'_b), it would place probability of 1 that the firm is bad. The most the lender could get by accepting is $\varepsilon \min[r'_b, R^g(1-\hat{x}_b)] + (1-\varepsilon)\min[r'_b, R^b(\hat{x}_b)] \approx \min[r'_b, R^b(\hat{x}_b)]$ for small ε, while by rejecting the offer the lender would receive $\varepsilon L^g + (1-\varepsilon)L^b \approx L^b$ for small ε. Since $\min[r'_b, R^b(\hat{x}_b)] < L^b$, the lender would reject the offer. So the bad firm's profit would be 0. If the bad firm complied, then it would receive $\pi^b(r^n, x^n) = \varepsilon \max[R^g(1-x^n) - r^n, 0]$ (since our assumption in footnote 22, that $R^b(1) < (d - p\theta L^g)/(1-p\theta)$, implies $R^b(x^n) < r^n$). Since this profit is greater than zero, the bad firm would prefer to comply than to make an offer that differs from the good firm's offer. Thus, there can be no separating equilibrium.

APPENDIX 3: PROOFS OF ASSERTIONS IN SECTION 4 ON EFFECTS OF RELAXING ASSUMPTION (10)

LEMMA 1. *There exists a critical value $p^* \in [0, 1]$ such that*

$$pR^g(1 - x^*(p)) + (1-p)R^b(x^*(p)) \geq (<)\, pL^g + (1-p)L^b \text{ iff } p \geq (<)\, p^*. \tag{A3.1}$$

Proof. Define:

$$\phi(p) \equiv pR^g(1 - x^*(p)) + (1-p)R^b(x^*(p))$$

and

$$L(p) \equiv pL^g + (1-p)L^b.$$

Observe that:

$$\phi(0) - L(0) = R^b(1) - L^b < 0 \text{ by assumption (8) in the text and}$$

$$\phi(1) - L(1) = R^g(1) - L^g > 0 \text{ by assumption (9) in the text.}$$

Therefore, to show (A3.1), all we need to show is that $\phi(p) - L(p) \geq 0$ implies $\phi'(p) - L'(p) \geq 0$.

Now:

$$\phi(p) - L(p) \geq 0 \Leftrightarrow R^g(1 - x^*(p)) - L^g \geq \frac{1-p}{p}[L^b - R^b(x^*(p))].$$

And:

$$\begin{aligned}\phi'(p) - L'(p) &= [R^{g}(1 - x^*(p)) - L^{g}] - [R^{b}(x^*(p)) - L^{b}] \\ &\quad + [-pR^{g}_{x}(1 - x^*(p)) + (1 - p(R^{b}_{x}(x^*(p))] \frac{dx^*}{dp} \\ &= [R^{g}(1 - x^*(p)) - L^{g}] - [R^{b}(x^*(p)) - L^{b}],\end{aligned}$$

using the first-order condition for the nonrenegotiable contract, Eq. (17) in the text. Therefore, $\forall\, p$ such that $\phi(p) - L(p) \geq 0$,

$$\begin{aligned}\phi'(p) - L'(p) &\geq \frac{1 - p}{p} [L^{b} - R^{b}(x^*(p))] - [R^{b}(x^*(p)) - L^{b}] \\ &= \frac{1}{p} [L^{b} - R^{b}(x^*(p))] \\ &\geq 0 \qquad \text{by assumption (8) in the text.} \qquad \text{Q.E.D.}\end{aligned}$$

LEMMA 2. *In the uninformed state, only pooling equilibria are possible when* $[\phi(p) - L(p))] > 0$.

Proof. Identical to the proof when $[\phi(p) - L(p)] < 0$ given in Appendix 2.

LEMMA 3. *In the uninformed state, if* $[\phi(p) - L(p)] > 0$ *and* $d + k > L(p)$, *then both types choose* $x^{g}_{j} = x^{b}_{j} = x'' < x^{n}$ *in period* 1 *and the contract is renegotiated.*

Proof. Given Lemma 2, we restrict attention to pooling equilibria. Assume that the borrowers choose $x^{g}_{j} = x^{b}_{j} = x'' < x^{n}$ in period 1 and offer contract (r'', x''), where r'' is the minimum r that will be accepted by the lender. That is, r'' satisfies

$$pr'' + (1 - p)R^{b}(x'') = L(p). \tag{A3.2}$$

A necessary and sufficient condition for this pooling equilibrium to exist is that the good firm prefers (x'', r'') to (x^{n}, r^{n}), which requires that

$$D(p) \equiv [R^{g}(1 - x'') - r''] - [R^{g}(1 - x^{n}) - r^{n}] > 0. \tag{A3.3}$$

It is easy to show that there is a unique x^* satisfying

$$x^* = \underset{x}{\operatorname{argmax}}[R^{g}(1 - x'') - r''] - [R^{g}(1 - x^{n}) - r^{n}] \quad \text{subject to (A3.2).}$$

So if $D(p)$ evaluated at $x'' = x^*$ is > 0, then the pooling equilibrium will exist. To show this, differentiate $D(p)$ evaluated at $x'' = x^*$ with respect to θ:

$$\frac{dD(p)}{d\theta} = R_x^g(1 - x^n)\frac{dx^n}{d\theta} + \frac{dr^n}{d\theta}. \tag{A3.4}$$

Recall that r^n solves

$$p[\theta L^g + (1 - \theta)r^n] + (1 - p)R^b(x^n) - d - k = 0.$$

Therefore,

$$\frac{dD(p)}{d\theta} = \left[R_x^g(1 - x^n) - \frac{1 - p}{p(1 - \theta)} R_x^b(x^n)\right]\frac{dx^n}{d\theta} + \frac{d + k}{p(1 - \theta)^2}$$
$$- \frac{1 - p}{p(1 - \theta)^2} R^b(x^n) - \frac{\theta L^g}{(1 - \theta)^2} - \frac{L^g}{1 - \theta}.$$

By the first-order condition for the renegotiable contract (Eq. (21) in the text), the first term is zero. Therefore,

$$\frac{dD(p)}{d\theta} = \frac{1}{p(1 - \theta)^2}[d + k - pL^g - (1 - p)R^b(x^n)].$$

So:

$$\text{sign}\left\{\frac{dD(p)}{d\theta}\right\} = \text{sign}\left\{\frac{1}{p(1 - \theta)^2}[d + k - pL^g - (1 - p)R^b(x^n)]\right\}.$$

Since, by assumption, $d + k > L(p) \equiv pL^g + (1 - p)L^b > pL^g + (1 - p)R^b(x^n)$, then $dD(p)/d\theta > 0$. Therefore, if $D(p)$ evaluated at $x'' = x^*$ is greater than 0 at $\theta = 0$, then $D(p)$ evaluated at $x'' = x^*$ is greater than 0 at all θ. To show $D(p; x'' = x^*) > 0$ at $\theta = 0$, note that at $\theta = 0$, $x^n = x^*$ and $r^n > r''$ (since by assumption, $d + k > L(p)$). So $D(p; x'' = x^*)$ at $\theta = 0$ is $r^n - r'' > 0$.

Thus, $(r'' = L(p)/p - ((1 - p)/p)\, R^b(x''),\ x'' = x^*)$ is a pooling equilibrium. Q.E.D.

PROPOSITION. *If $\phi(p) > L(p)$ and $d + k > L(p)$, then the renegotiable contract is infeasible.*

Proof. Given Lemmas 1–3, we know that if $\phi(p) > L(p)$ and $d + k > L(p)$, the lender's expected profit in period zero is

$$p[\theta L^g + (1 - \theta)r''] + (1 - p)[\theta R^b(x^n) + (1 - \theta)R^b(x'')] - (d + k)$$
$$= \theta[pL^g + (1 - p)R^b(x^n)] + (1 - \theta)[pr'' + (1 - p)R^b(x'')] - (d + k)$$

$$= \theta[pL^g + (1-p)R^b(x^n)] + (1-\theta)[L(p) - (1-p)R^b(x'') + (1-p)R^b(x'')] - (d+k) \qquad \text{by (A3.2)}$$

$$= \theta[pL^g + (1-p)R^b(x^n)] + (1-\theta)L(p) - (d+k)$$

$$< \theta[pL^g + (1-p)L^b] + (1-\theta)L(p) - (d+k)$$

$$= \theta L(p) + (1-\theta)L(p) - (d+k)$$

$$= L(p) - (d+k)$$

< 0 by assumption.

Thus, the contract is infeasible. Q.E.D.

APPENDIX 4: EXAMPLES

A. *Negative Exponential Revenue Functions*

Assume that

$$R^j(x) = k_j - \exp\{-b_j x\}, \qquad j = g, b, \tag{A4.1}$$

which satisfy assumption (2) in the text and also satisfy (6) and (7) through an appropriate choice of k_g and k_b. (Note that this is without further loss of generality since both k_g and k_b cancel when π^d is evaluated.)

An attractive feature of this example is that the optimal covenants may be calculated explicitly:

$$x^n = \frac{b_b + ln\{\phi\} - ln\{1-\theta\}}{b_g + b_b}, \tag{A4.2}$$

and

$$x^* = \frac{b_g + ln\{\phi\}}{b_g + b_b}, \qquad \text{where } \phi \equiv \frac{(1-p)b_b}{pb_g}. \tag{A4.3}$$

Using (A4.2) and (A4.3) it is easy to calculate the difference between the two covenant restrictions:

$$x^n - x^* = \frac{-ln\{1-\theta\}}{b_g + b_b}. \tag{A4.4}$$

Note that the difference between the optimal covenants is independent of p, an artifact of this particular functional form. So $dx^n/dp = dx^*/dp$. To check whether π^d is quasiconcave over the relevant region, we differentiate Eq. (24) in the text again with respect to p and use first-order conditions (17) and (21) to find

$$\frac{d^2\pi^d}{dp^2} = \frac{1}{p}\left\{[R^{\mathrm{b}}_x(x^*)]\frac{dx^*}{dp} - [R^{\mathrm{b}}_x(x^n)]\frac{dx^n}{dp}\right\}. \tag{A4.5}$$

Since $x^* < x^n$ and $R^{\mathrm{b}}(\cdot)$ is concave, $R^{\mathrm{b}}_x(x^*) > R^{\mathrm{b}}_x(x^n) > 0$. Recall that $dx^*/dp < 0$ and $dx^n/dp < 0$. Therefore, a sufficient condition for π^d to be concave (that is, $d^2\pi^d/dp^2 < 0$) is $|dx^*/dp| \geq |dx^n/dp|$. And since $dx^n/dp = dx^n/dp$ in this example, π^d is concave over the region with interior solutions and, therefore, satisfies the conditions of Proposition 4.

B. *Quadratic Revenue Functions*

Next, we examine the following quadratic revenue functions:

$$R^j(x) = k^j + ax - \tfrac{1}{2}bx^2, \qquad a > b, j = \mathrm{g}, \mathrm{b}, \tag{A4.6}$$

where k^j can be chosen to satisfy Eqs. (6) and (7) in the text without further loss of generality. The optimal covenants are

$$x^n = \frac{(1 - p)a - p(1 - \theta)(a - b)}{(1 - p\theta)b} \tag{A4.7}$$

and

$$x^* = \frac{(1 - p)a - p(a - b)}{b}. \tag{A4.8}$$

We now show that $\pi^d(\cdot)$ is quasiconcave in the relevant region. Differentiating (A4.7) and (A4.8) with respect to p yields

$$\frac{dx^n}{dp} = \frac{-(1 - \theta)(2a - b)}{(1 - p\theta)^2 b} \quad \text{and} \quad \frac{dx^*}{dp} = \frac{-(2a - b)}{b},$$

respectively. Substituting these and Eqs. (A4.7) and (A4.8) into Eq. (A4.5) yields

$$\begin{aligned}\frac{d^2\pi^d}{dp^2} &= -\frac{1}{p}\frac{(2a - b)}{b}\left\{a - [(1 - p)a - p(a - b)]\right.\\ &\quad \left. - \frac{1 - \theta}{(1 - p\theta)^2}\left(a - \frac{(1 - p)a - p(1 - \theta)(a - b)}{1 - p\theta}\right)\right\}\\ &= -\frac{1}{p}\frac{(2a - b)}{b}\left\{(2a - b)p - \frac{(1 - \theta)^2}{(1 - p\theta)^3}(2a - b)p\right\}\\ &= -\frac{(2a - b)^2}{b}\left\{1 - \frac{(1 - \theta)^2}{(1 - p\theta)^3}\right\}.\end{aligned} \tag{A4.9}$$

Hence, $d^2\pi^d/dp^2 < 0$ if and only if $p < (1 - (1 - \theta)^{2/3})/\theta$. Let $\hat{p} \equiv (1 - (1 - \theta)^{2/3})/\theta$. Remembering that p^c is the largest value of p for which both contracts yield interior solutions, then, if $p^c < \hat{p}$, $\pi^d(\cdot)$ is concave over the relevant region. If $p^c > \hat{p}$, then $d^2\pi^d(p)/dp^2 > 0$ for all p such that $\hat{p} < p \leq p^c$. As demonstrated in Proposition 3, $d\pi^d(p^c)/dp < 0$. Thus, over the region with interior solutions $[p^b, p^c]$, $\pi^d(\cdot)$ is quasiconcave.

REFERENCES

AGHION, P., AND BOLTON, P. (1988). "An Incomplete Contract Approach to Bankruptcy and the Financial Structure of the Firm," Technical Report No. 536. Institute for Mathematical Studies in the Social Sciences, Stanford University.

BARNEA, A., HAUGEN, R., AND SENBET, L. (1985). "Agency Problems and Financial Contracting." Prentice Hall, Englewood Cliffs, NJ.

BERLIN, M., AND LOEYS J. (1988). Bond covenants and delegated monitoring, *J. Finance* **43,** 397–412.

BLACKWELL, D. W., AND KIDWELL, D. S. (1988). An investigation of cost differences between public sales and private placements of debt, *J. Finan. Econ.* **22,** 253–278.

BROWN, D. T., JAMES, C. M., AND MOORADIAN, R. M. (1992). "The Information Content of Distressed Restructurings Involving Public and Private Debt Claims," Working Paper. Univ. of Florida.

BULOW, J. I., AND SHOVEN, J. B. (1978). The bankruptcy decision, *Bell J. Econ.* **9,** 437–456.

CASTLE, G. R. (1980). Term lending—A guide to negotiating term loan covenants and other financial restrictions, *J. Comm. Bank Lending* **63,** 26–39.

DEANGELO, H., DEANGELO, L., AND SKINNER, D. J. (1990). "An Empirical Investigation of the Relation between Accounting Choice and Dividend Policy in Troubled Companies," Working Paper No. 90-07. Univ. of Michigan School of Business Administration.

DIAMOND, D. (1990). "Seniority and Maturity Structure of Bank Loans and Publicly Traded Debt." Working Paper No. 308. Univ. of Chicago Center for Research in Security Prices.

GERTNER, R., AND SCHARFSTEIN, D. (1991). A theory of workouts and the effects of reorganization law, *J. Finance* **46,** 1189–1222.

HARRIS, M., AND RAVIV, A. (1990). Capital structure and the informational role of debt, *J. Finance* **45,** 321–349.

HART, O., AND MOORE, J. (1988). Incomplete contracts and renegotiation, *Econometrica* **56,** 755–787.

HART, O., AND MOORE, J. (1989). "Default and Renegotiation: A Dynamic Model of Debt," Working Paper. MIT.

HUBERMAN, G., AND KAHN, C. (1988a). Limited contract enforcement and strategic renegotiation, *Amer. Econ. Rev.* **78,** 471–484.

HUBERMAN, G., AND KAHN, C. (1988b). Strategic renegotiation, *Econ. Lett.* **28,** 117–121.

ISKANDAR, M. E., AND EMORY, D. R. (1990). "The Role of Indenture Provisions in Determining Bond Ratings," Working Paper. Bentley College.

JENSEN, M. C., AND MECKLING, W. H. (1976). Theory of the firm: Managerial behavior, agency costs, and ownership structure, *J. Finan, Econ.* **3,** 305–360.

JOHN, K., AND KALAY, A. (1982). Costly contracting and optimal payout constraints, *J, Finance* **37,** 457–470.

JOHN, K., AND KALAY, A. (1985). Informational content of optimal debt contracts, *in* "Recent Advances in Corporate Finance" (E. I. Altman and M. G. Subrahmanyam, Eds.), pp. 133–162. Richard D. Irwin, Illinois.

KAHN, C., AND HUBERMAN, G. (1989). Default, foreclosure, and strategic renegotiation, *Law Contemp. Probl.* **52,** 49–61.

KALAY, A. (1982). Stockholder–bondholder conflict and dividend constraints, *J. Finan. Econ.* **10,** 211–233.

LEFTWICH, R. (1981). Evidence of the impact of mandatory changes in accounting principles on corporate loan agreements, *J. Acc. Econ.* **3,** 3–36.

LUMMER, S. L., AND MCCONNELL, J. J. (1989). Further evidence on the bank lending process and the capital-market response to bank loan agreements, *J. Finan. Econ.* **25,** 99–122.

RAJAN, R. (1991). "Insiders and Outsiders: The Choice between Relationship and Armslength Debt," Working Paper. MIT.

ROSENBERG, R. M. (1984). An overview of workouts from the perspective of the institutional lender, *Loyola Univ. Law J.* **16,** 1–41.

SHARPE, S. (1988). "Dept Maturity Choice," Working Paper. Board of Governors, Federal Reserve System.

SMITH, C. W., AND WARNER, J. B. (1979). On financial contracting: an analysis of bond covenants, *J. Finan. Econ.* **7,** 117–161.

STULZ, R. M. (1990). Managerial discretion and optimal financing policies, *J. Finan, Econ.* **26,** 3–27.

STULZ, R. M., AND JOHNSON, H. (1985). An analysis of secured debt, *J. Finan. Econ.* **14,** 159–179.

WHITE, M. J. (1980). Public policy towards bankruptcy: Me-first and other priority rules, *Bell J. Econ.* **11,** 550–564.

ZINBARG, E. (1975). The private placement loan agreement, *Finan. Analysts J.* **31,** 33–52.